A dynamic model to predict suspended particulate matter in lakes

J.M. Malmaeus*, L. Håkanson
Department of Earth Sciences, Villavägen 16, S-752 36 Uppsala, Sweden
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Abstract
A dynamic model for predictions of suspended particulate matter (SPM) in lake water is presented. The model includes compartments for surface water, deep water and sediment areas of erosion/transportation. SPM pathways include tributary inflow, autochthonous production, outflow via rivers, sedimentation on erosion/transportation areas, sedimentation on accumulation areas, resuspension, mineralization and mixing. The model is driven by easily accessible lake variables, including morphometric parameters (like mean depth and lake area), climatic variables (like mean monthly wind from meteorological tables) and phosphorus inflow (from measurements or monitoring programs). The model has been tested against empirical data from a number of European lakes, with generally close agreement to empirical data. Uncertainty and sensitivity tests are performed to identify the most important model uncertainties. The difficulties in quantifying allochthonous and autochthonous SPM generation far outrank the problems associated with internal processes like, e.g. sedimentation and mineralization.

Keywords: Suspended particulate matter; Autochthonous material; Allochthonous material; Lakes; Dynamic model; Monte Carlo simulations

1. Introduction

The importance of suspended particulate matter (SPM) in limnological systems is very great. It is generally regarded as one major factor regulating water quality. Many papers and textbooks have discussed transport routes of nutrients and toxic substances directed by the presence of SPM (e.g. Håkanson, 1999), SPM as an energy source (e.g. Wetzel, 2001) and the influence of SPM on Secchi depth and primary production (e.g. Håkanson and Bouillon, 2002).

SPM in lake water includes material supplied by tributaries (allochthonous material), material produced in the water column (autochthonous material) and resuspended material (Håkanson and Peters, 1995). The chemical composition of SPM varies significantly among and within lakes. Many types of substances may be present, including clay minerals, humic substances, bacterial colonies, living and dead plankton, and detritus (Gustafsson and Gschwend, 1997). Operationally, SPM is often determined as being the non-filterable fraction in a water sample. To this end, filter pore sizes ranging between 0.2 and 0.9 μm (most commonly 0.45 μm) are used (e.g. Johansson et al., 2001). Using data on SPM concentrations from different lakes, measured with different techniques (and different filter pore sizes), may bias any attempt to produce or calibrate practically useful models. In this work, we follow the same approach as Gustafsson and Gschwend (1997), defining SPM

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as suspended particles that are subject to gravitational settling.

Despite the achieved knowledge and extended research, and the recognition of SPM as a key parameter in many contexts, it remains extremely difficult to predict concentrations of SPM in lakes and rivers. There are severe difficulties associated with quantifying the different sources of SPM, and the aim here is to highlight the uncertainties associated with SPM predictions in lakes and reservoirs. Lindström et al. (1999) presented an empirical model to predict typical SPM values in lakes using total phosphorus concentration, pH and dynamic ratio. To the best of our knowledge, there are no existing general dynamic models for predicting SPM concentrations in lakes in the literature. Occasionally, suspended solids concentration has been modeled at specific sites using, e.g., wind speed and water velocity as driving variables (e.g. Aalderink et al., 1984; Lueettich et al., 1990; Somlyódy and Koncsos, 1991).

In this paper, we present a dynamic model that predicts monthly mean values of SPM concentration in lake water. The model is tested for Lake Erken (Sweden), Lake Balaton (Hungary), Lake Kinneret (Israel), and Lakes Miastro and Naroch (Belarus) with respect to model performance and sensitivity.

An early outline of the model was used for predicting the impact of water level fluctuations in Lake Kinneret by Håkanson et al. (2000). The model presented here is also used as an integral part of the LEEDS-model, which is a holistic model for predicting eutrophication effects in lake ecosystems (see Malmaeus and Håkanson, 2003).

2. The model

The basic model structure and several algorithms are inherited from the successfully validated radionuclides model presented in Håkanson (2000). A schematic presentation of the model structure is given in Fig. 1. There are compartments for SPM in (1) epilimnetic water, (2) hypolimnetic water, and (3) ET-sediments (i.e. areas of sediment erosion and transport). A-sediments (i.e. areas of sediment accumulation) receive a given fraction of the settling material. The SPM fluxes included are (1) inflow of SPM via tributaries, (2) inflow via precipitation, (3) SPM generation related to primary production, (4) outflow, (5) sedimentation, (6) resuspension, (7) mixing between deep water and surface water, and (8) mineralization of organic material. The necessary driving variables for the model are given in Table 1.

The calculation time in this version is one month and SPM is represented as gram dry weight (g dw). The different model variables and abbreviations are compiled in Table 2.
Table 1
The driving variables in the SPM-model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude (°N)</td>
<td>Vertical location of the lake.</td>
</tr>
<tr>
<td>Altitude (m above sea level)</td>
<td>Vertical distance from sea level.</td>
</tr>
<tr>
<td>Annual precipitation (mm)</td>
<td>Annual rainfall in the area.</td>
</tr>
<tr>
<td>Light (monthly mean daylight hours per day)</td>
<td>Monthly mean number of daylight hours.</td>
</tr>
<tr>
<td>Catchment area (m²)</td>
<td>Area of the catchment.</td>
</tr>
<tr>
<td>Lake area (m²)</td>
<td>Area of the lake.</td>
</tr>
<tr>
<td>Mean depth (m)</td>
<td>Average depth of the lake.</td>
</tr>
<tr>
<td>Max. depth (m)</td>
<td>Maximum depth of the lake.</td>
</tr>
<tr>
<td>TP in precipitation (g per month)</td>
<td>Total phosphorus in precipitation.</td>
</tr>
<tr>
<td>TP in tributaries (µg/l, monthly values)</td>
<td>Total phosphorus in tributaries.</td>
</tr>
<tr>
<td>TP in lake water (µg/l, monthly values)</td>
<td>Total phosphorus in lake water.</td>
</tr>
<tr>
<td>SPM in tributaries (mg/l, monthly values)</td>
<td>Suspended particulate matter in tributaries.</td>
</tr>
</tbody>
</table>

*If SPM in tributaries is not available, it can be predicted from TP concentration in tributaries.

Table 2
Abbreviations and dimensions of variables used in the SPM-model

<table>
<thead>
<tr>
<th>Morphometric parameters</th>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D = depth (m)</td>
<td></td>
<td>Mean depth of accumulation areas (m)</td>
</tr>
<tr>
<td>D_A</td>
<td></td>
<td>Mean depth of accumulation areas (m)</td>
</tr>
<tr>
<td>D_E</td>
<td></td>
<td>Mean depth of erosion and transportation areas (m)</td>
</tr>
<tr>
<td>D_m</td>
<td></td>
<td>Maximum depth (m)</td>
</tr>
<tr>
<td>D_Z</td>
<td></td>
<td>Critical depth (m)</td>
</tr>
<tr>
<td>D_α</td>
<td></td>
<td>Depth of photic zone (m)</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>Lake area (m²)</td>
</tr>
<tr>
<td>A_ex</td>
<td></td>
<td>Epilimnetic area (m²)</td>
</tr>
<tr>
<td>V</td>
<td></td>
<td>Lake volume (m³)</td>
</tr>
<tr>
<td>V_a</td>
<td></td>
<td>Epilimnetic volume (m³)</td>
</tr>
<tr>
<td>V_p</td>
<td></td>
<td>Passive volume (m³)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Form parameters</th>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_d</td>
<td>form factor</td>
<td>Volume development (Vd = D_m/D_max)</td>
</tr>
<tr>
<td>DR</td>
<td>Dynamic ratio</td>
<td>(DR = √(V / A))</td>
</tr>
</tbody>
</table>

Other parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>(m a.s.l.)</td>
<td>Vertical distance from sea level.</td>
</tr>
<tr>
<td>MAX</td>
<td></td>
<td>Maximum norm for altitude.</td>
</tr>
<tr>
<td>MIN</td>
<td></td>
<td>Minimum norm for altitude.</td>
</tr>
<tr>
<td>Lat</td>
<td></td>
<td>Latitude (°N)</td>
</tr>
<tr>
<td>Li</td>
<td></td>
<td>Min. norm for latitude.</td>
</tr>
<tr>
<td>LS</td>
<td></td>
<td>Max. norm for latitude.</td>
</tr>
<tr>
<td>Light factor</td>
<td></td>
<td>Number of daylight hours (h per day)</td>
</tr>
<tr>
<td>MAET</td>
<td></td>
<td>Mean annual epilimnetic temperature (°C)</td>
</tr>
<tr>
<td>PP</td>
<td></td>
<td>Particulate phosphorus fraction.</td>
</tr>
<tr>
<td>Prec</td>
<td></td>
<td>Annual precipitation (mm)</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td>River discharge (m³ per month)</td>
</tr>
<tr>
<td>QM</td>
<td></td>
<td>Mean annual discharge (m³)</td>
</tr>
</tbody>
</table>

2.1 Inputs

SPM is generated internally, through autochthonous production, and externally, from rivers and precipitation. All inflows are given in g dw per month.

At present, the parts predicting the input of SPM via inflows and SPM from primary production are not very well developed. According to our knowledge, no validated mechanistically-based dynamic models are available for predicting autochthonous SPM production on a monthly basis. Tributary input can be predicted, but with high uncertainty using, e.g. tributary total phosphorus, catchment characteristics and hydrological variables (Håkanson et al., 2003). This means that empirical data should be used if possible for the primary fluxes of SPM.

2.1.1 Allochthonous material

2.1.1.1 Tributaries. The allochthonous sediment yield from rivers could, in principle, be modeled with existing knowledge. However, even in rather specialized studies, utilizing GIS processing of catchment properties such as soil properties, slope and drainage density (e.g. Verstraeten and Poesen, 2001), prediction of sediment yield remains rather imprecise. Apparently, the link between soil loss and sediment transport is problematic, as is the spatial and temporal lumping of catchment characteristics (Walling, 1983; Atkinson, 1995). These issues are further addressed in Håkanson et al. (2003).
where $\mathbf{A} = \mathbf{Q} \times \mathbf{P}$ is river discharge in m³ per month. $\mathbf{Q}$ is predicted by a sub-model originally presented in Abrahamsson and Håkanson (1998). Mean annual discharge ($\mathbf{Q}_A$) is given by

$$\mathbf{Q}_A = \mathbf{A}_{DA} \left( \frac{\text{Prec}}{307} \right) \times 0.01 \times 60 \times 60 \times 24 \times 365$$  

(2)

where $\mathbf{A}_{DA}$ is the catchment area (km²). In Eq. (2), the annual precipitation (Prec, mm per year) is related to a standard precipitation (650 mm) and multiplied by a general specific runoff rate (0.01 m²/km² s) and conversion factors into m³ per year. Monthly mean values on tributary discharge are predicted by means of a dimensionless moderator ($Y_Q$) so that

$$Q = \frac{Y_Q \times Q_A}{12}$$  

(3)

$Y_Q$ is calculated each month as

$$Y_Q = 1 + \left[ \frac{0.526(\text{Lat} - 35)^{2.18}}{352.18} \times L_x \right]$$

$$+ \left[ 1 - (\text{Lat} - 35)^{2.18} \times L_i \right]$$

$$+ \left[ \frac{0.421(\text{Alt})^{0.51}}{1000^{0.51}} \times A_x \right]$$

$$+ \left[ \frac{0.265(Q_A)/(60 \times 60 \times 24 \times 365)^{0.22}}{5000^{0.22}} \times Q_x \right]$$

$$+ \left[ 1 + (Q_A/(60 \times 60 \times 24 \times 365)^{0.22} \times Q_i \right]$$

(4)

Latitude (Lat, °N) and altitude (Alt, m a.s.l.) are required as input variables. Norm values meant to describe monthly variability in flow patterns due to latitude, altitude and mean annual discharge are used in this sub-model. The norms, $L_x$, $L_i$, $A_x$, $A_i$, $Q_x$, and $Q_i$ are defined in Table 3.

Table 3  
Seasonal norms for water discharge (max. norm = $Q_A$, min. norm = $Q_i$), latitude (max. norm = $L_x$ and min. norm = $L_i$), and altitude (max. norm = $A_x$ and min. norm = $A_i$)

<table>
<thead>
<tr>
<th>Month</th>
<th>$Q_A$</th>
<th>$Q_i$</th>
<th>$L_x$</th>
<th>$L_i$</th>
<th>$A_x$</th>
<th>$A_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.71$</td>
<td>$0.58$</td>
<td>$-1$</td>
<td>$1.04$</td>
<td>$-0.97$</td>
<td>$0.47$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.48$</td>
<td>$0.81$</td>
<td>$-1$</td>
<td>$1.37$</td>
<td>$-0.98$</td>
<td>$0.51$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.17$</td>
<td>$0.84$</td>
<td>$-1$</td>
<td>$0.56$</td>
<td>$-0.58$</td>
<td>$0.22$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.17$</td>
<td>$1.58$</td>
<td>$-1$</td>
<td>$0.38$</td>
<td>$-0.69$</td>
<td>$0.24$</td>
</tr>
<tr>
<td>5</td>
<td>$0.62$</td>
<td>$-0.1$</td>
<td>$2.17$</td>
<td>$-0.29$</td>
<td>$2.11$</td>
<td>$0.18$</td>
</tr>
<tr>
<td>6</td>
<td>$1.74$</td>
<td>$-1$</td>
<td>$2.51$</td>
<td>$-0.23$</td>
<td>$1.87$</td>
<td>$-0.52$</td>
</tr>
<tr>
<td>7</td>
<td>$0.32$</td>
<td>$-1$</td>
<td>$0.63$</td>
<td>$-0.62$</td>
<td>$0.51$</td>
<td>$-0.42$</td>
</tr>
<tr>
<td>8</td>
<td>$0.09$</td>
<td>$-1$</td>
<td>$0.24$</td>
<td>$-0.71$</td>
<td>$0.07$</td>
<td>$-0.49$</td>
</tr>
<tr>
<td>9</td>
<td>$-0.16$</td>
<td>$-0.82$</td>
<td>$0.05$</td>
<td>$-0.79$</td>
<td>$0.03$</td>
<td>$-0.38$</td>
</tr>
<tr>
<td>10</td>
<td>$-0.20$</td>
<td>$-0.56$</td>
<td>$-0.03$</td>
<td>$-0.74$</td>
<td>$-0.06$</td>
<td>$-0.20$</td>
</tr>
<tr>
<td>11</td>
<td>$-0.63$</td>
<td>$0.11$</td>
<td>$-0.66$</td>
<td>$-0.28$</td>
<td>$-0.62$</td>
<td>$0.07$</td>
</tr>
<tr>
<td>12</td>
<td>$-0.44$</td>
<td>$0.54$</td>
<td>$-0.92$</td>
<td>$0.32$</td>
<td>$-0.68$</td>
<td>$0.13$</td>
</tr>
</tbody>
</table>

From Ottosson and Abrahamsson (1998).
2.1.1.2 Precipitation. In some areas, the SPM content in the precipitation may be high. As for the tributary inflow, the SPM concentration is thought to be related to the phosphorus concentration in the water. Also, a dimensionless moderator describing the wind influence on airborne dust is included:

\[ F_{\text{In, Prec}} = \frac{Y_{\text{Wind}} \times TP_{\text{Prec}}}{0.0203} \]  

(5)

The TP precipitation (TP Prec) is a model driving variable, given in g TP per month. The wind moderator (Y Wind) is given by

\[ Y_{\text{Wind}} = \left( \text{Wind}^{3.27} \right)^{2} \]  

(6)

where Wind is the mean wind speed a given month (m/s). Monthly mean values on wind speed are required as model input data. 3.27 is a characteristic mean monthly wind velocity for central Sweden.

2.1.2 Autochthonous material

Many authors have discussed the predictability of the primary production of organic carbon in lakes. Generally, mean summer chlorophyll concentration is predicted from light availability and nutrient status (e.g. Dillon and Rigler, 1974; Smith, 1979; Riley and Prepas, 1985; Evans et al., 1996). Equations to accurately describe the amount of SPM generated on a monthly basis in a given lake are not available today, as far as we know. In this model, the autochthonous production is assumed to be related to the primary phytoplankton production (PP; g dw per month) such that

\[ F_{\text{In, Prod}} = k \times PP \]  

(7)

The constant, \( k \), is lake specific. Apparently, different food-webs and plankton species will prevail in different lakes, implying that the production of SPM is not a simple function of primary phytoplankton production. Phytoplankton, herbivorous zooplankton grazing on phytoplankton, bacterioplankton, benthic algae and detritus from flora and fauna, all contribute to SPM in a lake. For example, bacterioplankton may derive their carbon from dissolved organic matter, hence contributing to autochthonous SPM production, which is not taken into account when primary production is measured by traditional methods (Boulion and Håkanson, 2003). At this stage, \( k \) has to be calibrated against empirical data on phytoplankton.

The primary production is estimated with an equation modified from Smith (1979):

\[ PP = 0.001 \left( 10 \times C_{\text{TP}} - 79 \right) \left( 1 - \frac{1 - \text{PF}}{0.5} \right) \times Y_{\text{Light}} \left( \frac{\text{EpiTemp}}{\text{MAET}} \right) \times \frac{A \times D_{z} \times 30}{0.45} \]  

(8)

where \( C_{\text{TP}} \) is the phosphorus concentration in the lake (μg/l). The original equation has been complemented with

1. a moderator for the dissolved fraction \( 1 - \text{PF} \) compared with a reference value (0.5) where PF is calculated as (Johnson and Håkanson, 2002):

\[ \text{PF} = 0.12 \times \log(C_{\text{SPM}}) + 0.71 \]  

(9)

The concentration of SPM \( (C_{\text{SPM}}, \text{mg/l}) \) is the target variable of the dynamic model and is, as such, continuously recalculated.

2. a light moderator, \( Y_{\text{Light}} \), relating the number of hours with daylight (Light factor, h per day, monthly mean values) to a norm value (11.2 h per day):

\[ Y_{\text{Light}} = 1 + 0.2 \left( \frac{\text{Light factor}}{11.2} - 1 \right) \]  

(10)

Monthly mean values on hours with daylight are required as model input data. The amplitude value is 0.2. This means that PP will decrease by a factor of 0.9 if the number of hours with daylight is halved.

3. a temperature moderator relating the mean monthly epilimnetic temperature (EpiTemp, °C) to mean annual epilimnetic temperature (MAET, °C). Temperature values are calculated by a sub-model presented in Ottosson and Abrahamsson (1998). The temperature sub-model requires latitude (°N), altitude (m a.s.l.), continentality (km from ocean), lake mean depth \( (D_{m}, \text{m}) \) and lake volume \( (= \text{mean depth} \times \text{lake area}, \text{m}^{3}) \) as driving variables.

There are unit conversion factors transforming the primary production into dry weight (the factor 0.45) and monthly figures (the factor 30). Multiplying with lake area \( (A, \text{m}^{2}) \) and the depth of the photic zone \( (D_{z}, \text{m}) \).
m) gives the gross lake production. The depth of the photic zone is taken as the Secchi depth \((D_{\text{Secchi}}, \text{m})\). If the Secchi depth exceeds the mean depth, the latter is taken as the depth of the photic zone. Lake area and mean depth are required as input data and the Secchi depth is calculated as (Håkanson and Boulion, 2002):

\[
D_{\text{Secchi}} = 10^{-0.698 \times \log_{10}(1 + C_{\text{SPM}}) + 0.769}
\]

(11)

### 2.2. Internal processes

#### 2.2.1. Water compartments

**SPM in surface water.** The fluxes of SPM to and from the surface water compartment are given by the following equation:

\[
M_{\text{SW}}(t) = M_{\text{SW}}(t - \Delta t) + (F_{\text{In, Ri}} + F_{\text{In, Prec}} + F_{\text{In, Prod}} + F_{\text{ETSW}} - F_{\text{SWDWA}} - F_{\text{SWT}} - F_{\text{SWDWS}} - F_{\text{SWm}}) \Delta t
\]

(12)

where

- \(M_{\text{SW}}\): mass (= amount) of SPM in the surface water (g dw);
- \(F_{\text{In, Ri}}\): flow of SPM (g per month) into the lake from tributaries (from Eq. (1));
- \(F_{\text{In, Prec}}\): flow of SPM (g per month) into the lake from precipitation (from Eq. (5));
- \(F_{\text{In, Prod}}\): allochthonous production of SPM (g per month) in surface water (from Eq. (7));
- \(F_{\text{ETSW}}\): mixing (x) of SPM from deep water (DW) to surface water (SW, g per month);
- \(F_{\text{SWDWA}}\): sedimentation (s) from surface water to deep water (g per month);
- \(F_{\text{SWT}}\): outflow of SPM from surface water to rivers (g per month);
- \(F_{\text{SWDWS}}\): mixing from surface water to deep water (g per month);
- \(F_{\text{SWm}}\): mineralization (m) of SPM in surface water (g per month).

**SPM in deep water.** The fluxes of SPM to and from the deep water compartment are given by the following equation:

\[
M_{\text{DW}}(t) = M_{\text{DW}}(t - \Delta t) + (F_{\text{SWDWS}} + F_{\text{SWDWx}} + F_{\text{ETDW}} - F_{\text{DWA}} - F_{\text{DWSWx}} - F_{\text{DWm}}) \Delta t
\]

(13)

where

- \(M_{\text{DW}}\): amount of SPM in deep water (g dw);
- \(F_{\text{SWDWS}}\): sedimentation from surface water to deep water (g per month);
- \(F_{\text{SWDWx}}\): mixing from surface water to deep water (g per month);
- \(F_{\text{ETDW}}\): resuspension from ET-areas to deep water (g per month);
- \(F_{\text{DWA}}\): sedimentation from deep water to A-areas (A, g per month);
- \(F_{\text{DWSWx}}\): mixing from deep water to surface water (g per month);
- \(F_{\text{DWm}}\): mineralization of SPM in deep water (g per month).

**2.2.1.1. Sedimentation. Surface water to ET-areas.**

The sedimentation flux from surface water to ET-areas is given by

\[
F_{\text{SWET}} = M_{\text{SW}} \times \text{ET} \times \frac{v}{D_{\text{ET}}} \times [(1 - D_{\text{sr}}) + 5 \times D_{\text{sr}}]
\]

(14)

where

- \(M_{\text{SW}}\): amount of SPM in the surface water (g dw);
- \(v\): fall velocity for SPM (m per month);
- \(D_{\text{sr}}\): distribution coefficient describing the fraction of resuspended SPM in surface water;
- \(\text{ET}\): fraction of ET-areas;
- \(D_{\text{ET}}\): the mean depth of the ET-areas.

The parameters in Eq. (14) are explained and discussed in the following:

A default value for the settling velocity of SPM is set to 5 m per year. This value is based on calibrations of the LEEDS-model for particulate phosphorus (Malmaeus and Håkanson, 2003). The value is modified by the SPM concentration such that

\[
v = \frac{1}{17} \times Y_{\text{SPM}}
\]

(15)
where
\[ Y_{SPM} = 1 + 3(C_{SPM} - 1) \]  
(16)

When the SPM-concentration is high the settling velocity is expected to increase since the material will aggregate into larger particles. The coagulation rate is a function of the total solids concentration (Gustafsson and Gschwend, 1997). According to Stokes’ law, smaller particles will be more sensitive to the viscosity of the medium and settle slower than larger ones. Low SPM concentrations may imply that a substantial fraction of larger particles have settled out, increasing the fraction of small particles with low average settling velocity in the water column (cf. Luettich et al., 1990). The constants in Eq. (16) originate from calibrations of the LEIDS-model.

The rate of sedimentation is further influenced by the amount of resuspended material. Resuspended particles are likely to have passed through benthic animals several times. This creates a gluing of the particles which are aggregated into larger flocs (Håkanson and Jansson, 1983). Resuspended particles also have a shorter distance to settle as compared to other particles in the lake water. Hence, these particles will have a higher sedimentation rate (Bloesch, 1995; Weyhenmeyer, 1998).

The resuspended fraction of SPM in surface water is calculated in the model as
\[ D_{Crs} = \frac{F_{ETSW}}{F_{FSW,Prod + F_{FSW,Bi} + F_{FSW,Poc + F_{ETSW}}} \]  
(17)

The resuspended material settles five times faster than the primary materials (the factor 5 in Eq. (14); it is based on calibrations).

A typical figure for the actual average settling velocity, if the resuspended fraction of SPM is taken as 30% and the SPM concentration is taken as 5 mg/l, would be around 140 m per year. This is substantially lower than values generally obtained in both field and laboratory conditions (see, e.g. Burban et al., 1990; Somlyody and Koncsos, 1991). However, it should be noted that most experiments are performed with a given SPM composition in a given environment, which may not represent the conditions in a whole lake.

Living phytoplankton and zooplankton contribute to SPM but not to sedimentation in the same manner as dead matter. Further, it is possible that sediment traps overestimate settling velocity since near bottom turbulence is dissipated (Aalderink et al., 1984).

The settling material is distributed between ET-areas and A-areas. The fraction of ET-areas (ET) is calculated from a sub-model given in Håkanson (1999). To get the rate of sedimentation on ET-areas, the settling velocity is divided by the depth of the water compartment \( D_{ET} \), which is calculated from the same sub-model. The sub-model for ET-areas is given in Fig. 2.

**Surface water to deep water.** The sedimentation flux from surface water to deep water is calculated in a similar manner as the sedimentation flux to the ET-areas (Eq. (14)), except that the factor ET is replaced by the A-factor \((1 - ET)\) and \( D_{ET} \) is replaced by the depth of the wave base \( D_{TA} \) determined from the sub-model outlined in Fig. 2. That is
\[ F_{SWDWs} = M_{SW} \times \frac{v}{D_{TA}} \times \left[ (1 - D_{Crs}) + 5 \times DC_{rs} \right] \]  
(18)

**Deep water to A-areas.** The sedimentation flux from deep water to A-areas is again very similar to Eqs. (14) and (18):
\[ F_{DWA} = M_{SW} \times \frac{v}{D_{TA}} \times \left[ (1 - D_{Crs}) + 5 \times DC_{rs} \right] \]  
(19)

The mean depth of the deep water compartment \( D_{A} \) is calculated in the same sub-model as \( D_{ET} \) and \( D_{TA} \) (see Fig. 2). The fraction of resuspended particulate matter in deep water is calculated as
\[ D_{Crd} = \frac{F_{ETDW}}{F_{ETDW} + F_{ETSW}} \]  
(20)

The gross sedimentation on A-areas (GS, g dw/m² per day) may be calculated as
\[ GS = \frac{F_{DWA}}{30 \times A \times (1 - ET)} \]  
(21)

Gross sedimentation is needed in many mass balance models for various substances to predict the burial rate and the substance concentration in the sediments.

**2.2.1.2. Mixing.** The mixing between deep water and surface water depends on the thermal stratification between the water masses, modified with a
Fig. 2. The sub-model to predict ET-areas and depth of wave base. The shaded variables are obligatory driving variables. Modified from Håkanson (1999).

The sub-model to predict ET-areas and depth of wave base: The shaded variables are obligatory driving variables. Modified from Håkanson (1999).


dimensionless moderator for mean monthly wind speed. The variable describing stratification (Strat) is calculated as:

\[
\text{If } |EpiTemp - HypoTemp| < 5 \text{ }^\circ\text{C} \text{ then } \text{Strat} = 1
\]

The surface water and deep water temperatures, EpiTemp and HypoTemp (\(^\circ\text{C}\)), are calculated by a sub-model presented by Ottosson and Abrahamsson (1998). The temperature sub-model requires latitude (\(^\circ\text{N}\)), altitude (m a.s.l.), continentality (km from ocean), lake mean depth (\(D_m\), m), and lake volume (\(V = A D_m\), m\(^3\)) as driving variables. If the difference between epilimnetic and hypolimnetic temperatures is smaller than 5 \(^\circ\text{C}\), it is assumed that the lake is not stratified and Strat is set to 1.

If the dynamic ratio (\(DR = \sqrt{(A \times 10^{-6})/D_m}\)) is higher than 3.8 (like, e.g. in Lake Balaton) the lake is probably not dimictic but polymictic, since it is comparatively large and shallow (Håkanson and Jansson, 1983). A rule for this is added to Eq. (22):

\[
\text{If } DR > 3.8 \text{ then } \text{Strat} = 1
\]

\[
\text{Surface water to deep water.} \quad \text{The mixing from surface water to deep water is given by}
\]

\[
\text{If Strat} = 1 \text{ then } F_{SW\text{\rightarrow}DW} = 2 \times M_{SW}\text{ else } F_{SW\text{\rightarrow}DW} = M_{SW} \times \text{Strat} \times V_{\text{wind}}
\]

When there is no stratification, the amount of SPM in the surface water is exchanged with the deep water.
twice per month, otherwise the mixing flux is smaller. The wind moderator is the same as for SPM precipita-
tion, given by Eq. (6).

Deep water to surface water. The mixing from deep
to the surface water is given by

\[ \text{If } \text{Strat} = 1 \text{ then } F_{DWSW} = 2 \times M_{DW} \]
\[ \text{else } F_{DWSW} = M_{DW} \times (V_{active}/V_{passive}) \]  

\[ \times \text{Strat} \times Y_{Wind} \]  

Eq. (25) differs from Eq. (24) in that the transported
material should be proportional to the exchanged water
volume. This is not accounted for when the lake is
mixed. The active volume is calculated as

\[ \text{If } V_{epi} < 0.5 \times A \text{ then } V_{active} = 0.5 \times A \]
\[ \text{else } V_{active} = V_{epi} \]  

The depth of the active volume is always larger than
0.5 m. The epilimnetic volume, \( V_{epi} \), is calculated as

\[ \text{If } D_{epi} = 0 \text{ then } V_{epi} = D_{epi} \times A_{epi} \]
\[ \text{else } V_{epi} = (A - A_{epi}) \times D_{epi}/3 + A_{epi} \times D_{epi} \]  

where

\[ A_{epi} = A \times 0.01 \]
\[ \times \left( \frac{100 \exp(-3 \times Vd^{1/3}) \times (D_{active} - D_{epi})^2}{D_{epi}^{1/3} + D_{max}^{1/3} \exp(-3 \times Vd^{1/3})} \right)^{0.5/Vd} \]  

\[ \times 10^{(2 - 1/Vd)} \]  

and

\[ D_{epi} = 0.25 \times \text{ABS(EpiTemp - HypoTemp)} \]  

Vd is called the volume development, or the form
factor, and is defined as \( D_{epi}/D_{max} \). It is a factor related to
the hypographic curve of a lake (Håkanson and
Jansson, 1983).

These calculations are based on Håkanson (1999).
The passive volume is simply the difference between
the lake volume and the active volume:

\[ V_{passive} = V - V_{active} \]  

2.2.1.3. Mineralization. Particulate matter is miner-
alized by bacteria in the water column, and the process
is temperature dependent (Tornblom and Pettersson,
1998). The rate of mineralization in the epilimnion
\( (R_{m,\text{epi}}) \) and hypolimnion \( (R_{m,\text{hypo}}) \) is given by

\[ R_{m,\text{epi}} = R_{m,\text{detail}} \times \frac{\text{EpiTemp}}{4} \]
\[ R_{m,\text{hypo}} = R_{m,\text{detail}} \times \frac{\text{HypoTemp}}{4} \]  

respectively, where the default rate of mineralization,
\( R_{m,\text{detail}} \), after calibrations is set to 0.6 per month;
4 °C is a reference temperature. The temperatures in
the epilimnion and in the hypolimnion are determined in
the temperature sub-model. The fluxes of SPM from
the two compartments are calculated as

\[ F_{SWm} = M_{DW} \times R_{m,\text{epi}} \]  

\[ F_{SWm} = M_{DW} \times R_{m,\text{hypo}} \]  

2.2.1.4. Outflow. It is assumed that outflow only oc-
curs from the surface water compartment and the flux
is given by

\[ F_{out} = M_{SW} \times M_{out} \times Y_{Q} \times \text{Evaporation}_\text{coefficient} \]  

where \( M_{out} \) is the outflow rate given by (Håkanson,
1999):

If \( T_{w} \times 12 < 1 \) then \( M_{out} = 1.386/(T_{w} \times 12) \) else
If \( A < 0.2 \times 10^{-4} \) then
\[ M_{out} = 1.386/(T_{w} \times 12) \times (0.5)/(0.5 + 1 + 0.5)/5 \]
else
\[ M_{out} = 1.386/(T_{w} \times 12) \times (0.5)/(0.5 + 1 + 0.5)/5 \]

\[ T_{w} \] is the theoretical water retention time calculated as

\[ T_{w} = \frac{V}{Q_{A}} \]  

where \( V \) is the lake volume (m 3 ) and \( Q_{A} \) is the mean
annual water discharge given by Eq. (2).

\( Y_{Q} \) in Eq. (35) is the seasonal moderator for water
discharge given by Eq. (4). The evaporation coefficient
is given by

\[ \text{Evaporation}_\text{coefficient} = 1 \]
\[ \text{Evaporation}_\text{coefficient} = 0 \]  

\[ \text{Evaporation}_\text{coefficient} = -1.67 \times (\text{MAET}/25 - 1) \]  

\[ \text{Evaporation}_\text{coefficient} = -1.66 \times (\text{MAET}/25 - 1) \]  

\[ \text{Evaporation}_\text{coefficient} = -1.66 \times (\text{MAET}/25 - 1) \]
This means that the outflow depends linearly on the mean annual epilimnetic temperature (MAET) between 10 and 25 °C. If MAET > 25 °C there is no outflow, due to high evaporation. If MAET < 10 °C there is no influence of evaporation.

2.2.2. Sediment compartment ET-areas. The fluxes of SPM to and from this compartment are given by the following equation:

\[ M_{ET}(t) = M_{ET}(t - dt) + (F_{SWET} - F_{ETDW} - F_{ETSW}) \cdot dt \]  (39)

where

- \( M_{ET} \): mass of SPM in the ET-sediments (g dw);
- \( F_{SWET} \): sedimentation from surface water to ET-areas (g per month);
- \( F_{ETDW} \): advective flow (resuspension) from ET-areas to deep water (g per month);
- \( F_{ETSW} \): advective flow from ET-areas to surface water (g per month).

The sedimentation flux from surface water to ET-areas is described in Section 2.2.1.1. The resuspension algorithms are described in the following section.

2.2.2.1. Resuspension. The form factor, \( V_d \), is used as a dimensionless distribution coefficient to regulate how much of the resuspended material will go to the surface water or to the deep water. The advective flux from ET-sediments to surface water is given by

\[ F_{ETSW} = M_{ET} \times R_{res} \left( \frac{1 - V_d}{3} \right) \]  (40)

The advection rate, \( R_{res} \), is given by

\[ R_{res} = \frac{\text{Wind}}{3.27^2} \]  (41)

where \( T_{ET} \) is the age of the ET-sediments, set to 12 months (from Håkanson, 1999). Wind is the mean wind speed a given month (m/s). The wind is related to a reference value of 3.27 m/s. Aalderink et al. (1984) and Somlyódy and Koncsos (1991) used relationships very similar to Eq. (41) for short term modeling of wind induced resuspension. According to Aalderink et al. (1984), the exponent in the equation should have a value between 1 and 3.

3. Model performance

The model has been run and compared with empirical data for Lake Erken (Sweden), Lake Balaton (Hungary), Lake Kinneret (Israel), Lake Miastro (Belarus) and Lake Naroch (Belarus). A summary of the lake characteristics is presented in Table 4. Most of the data on inflow and lake concentrations of SPM and phosphorus are unpublished, except for Lake Kinneret (see Håkanson et al., 2000). Lake Balaton is divided into four different sub-basins, with Basin 2 downstream from Basin 1, etc., as motivated by Somlyódy and van Straten (1986). For Lake Balaton and Lake Kinneret, empirical data on inflowing SPM have been used. For the other lakes, the inflow of total phosphorus has been used to estimate SPM inflow (see Section 2.1.1.1).

<table>
<thead>
<tr>
<th>Lake</th>
<th>Latitude (°N)</th>
<th>Altitude (m a.s.l.)</th>
<th>Annual precipitation (mm)</th>
<th>Catchment area (km²)</th>
<th>Lake area (km²)</th>
<th>Mean depth (m)</th>
<th>Max. depth (m)</th>
<th>Mean lake TP (µg/l)</th>
<th>Mean lake SPM (mg/l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Erken</td>
<td>59</td>
<td>11</td>
<td>660</td>
<td>140</td>
<td>24</td>
<td>9.0</td>
<td>21</td>
<td>27</td>
<td>2.3</td>
</tr>
<tr>
<td>Lake Miastro</td>
<td>55</td>
<td>150</td>
<td>650</td>
<td>130</td>
<td>13</td>
<td>5.4</td>
<td>11</td>
<td>40</td>
<td>4.1</td>
</tr>
<tr>
<td>Lake Naroch</td>
<td>47</td>
<td>106</td>
<td>600</td>
<td>280</td>
<td>80</td>
<td>9.0</td>
<td>25</td>
<td>22</td>
<td>1.6</td>
</tr>
<tr>
<td>Lake Balaton,</td>
<td>47</td>
<td>106</td>
<td>600</td>
<td>2750</td>
<td>38</td>
<td>2.3</td>
<td>4.5</td>
<td>87</td>
<td>27.0</td>
</tr>
<tr>
<td>Basin 1</td>
<td>144</td>
<td>2.9</td>
<td>4.1</td>
<td>41</td>
<td>76</td>
<td>24.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basin 2</td>
<td>47</td>
<td>106</td>
<td>600</td>
<td>2700</td>
<td>160</td>
<td>24</td>
<td>42</td>
<td>27</td>
<td>3.4</td>
</tr>
<tr>
<td>Lake Kinneret</td>
<td>32</td>
<td>−210</td>
<td>400</td>
<td>2000</td>
<td>180</td>
<td>24</td>
<td>42</td>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

*Total catchment area of Basin 2 includes the catchment area of Basin 1, i.e. the catchment area equals 4347 km².*
The model predictions compared to empirical data are given in Fig. 3. The calibration constant for primary production \( k \) is adjusted for each lake. It must be stressed that even though the mean values for the lake variables are derived from several years of empirical data, they should be considered uncertain. This is illustrated in Fig. 4, where the 95% confidence limits are shown for the empirical data from Lake Erken and Lake Balaton, Basin 2.

From the results given in Figs. 3 and 4, one can conclude that the model predictions are generally close to empirical data and within the uncertainty bands of

![Figures](image-url)
the empirical data, except for Lake Kinneret. In the following, we will discuss the model behavior in more detail.

4. Sensitivity tests

The sensitivity of the predictions to uncertainties in different model variables were examined using Monte Carlo simulations by varying the different fluxes and evaluating the effect on the model outcome. April and October were chosen for these sensitivity tests. Characteristic CV-values (coefficient of variation = standard deviation/mean value) were assigned to the different SPM fluxes and Monte Carlo simulations using 100 runs were performed for each $x$-variable.

Generally, the differences between lakes were much larger than the differences between months. The results for Lake Erken, Lake Balaton (Basin 2) and Lake Kinneret in October are presented in Fig. 5. For comparison the uncertainty in model outcome when all $x$-variables are varied simultaneously is also shown in the figure. One can note that the most important model uncertainties are generally, as expected, associated with the large fluxes. In our case primary production, sedimentation and SPM mineralization are the most important uncertainties inasmuch as they constitute the largest fluxes in the mass balance.

5. Discussion

In most lakes, the empirical data show a spring peak in April–May, which is not very well predicted by the model. This peak could be explained by high tributary influx of SPM or by high resuspension after the winter calm. The sensitivity analyses show, however, that both these processes impose minor influence on the SPM predictions in this model. It is also possible that the primary production of SPM is higher in the early spring than during the following months, possibly due to low predation pressure or due to the composition of total phosphorus (the dissolved phosphorus fraction may decrease during the growing season, see Reynolds, 1992).

The spring peak in Lake Kinneret coincides with an empirical peak in chlorophyll concentration. The model predicts a peak in late autumn due to high empirical phosphorus concentration. However, this phosphorus peak does not give any increase in productivity in reality. The Secchi depth is high during this period, so light should not be a limiting factor, neither temperature. The reason for the low primary production in late autumn may be biological interactions on a level not accounted for in this model. The spring peak in lake water appears some three months later than the peak in SMP inflow (see Håkanson et al., 2000), implying that primary production is much more important than the tributary inflow for the lake concentration of SPM in Lake Kinneret (which is well known).

The simulated SPM concentrations do not decrease very much during the winter months. This is not only true for Lake Kinneret, but also for the other lakes. In lakes with ice-cover the primary production should approach zero during the winter months, which is not the case in this model. Even the resuspension of particulate matter should be very low during the winter.
Fig. 5. Sensitivity tests using Monte Carlo simulations, 100 runs. The coefficient of variation (CV for $y$), i.e. the standard deviation divided by the mean value, is shown for (a) Lake Erken, (b) Lake Balaton, Basin 2, and (c) Lake Kinneret. The characteristic CV for all variables is 0.5 except for SPM outflow, which is set to 0.2. The sensitivity is compared to the uncertainty when all $x$-variables are varied at the same time (the leftmost box-and-whisker). Note that for Lake Balaton, the uncertainty in the SPM flux from the upstream basin is also included as a parameter.
Fig. 5. (Continued).

CV for x:
- 0.5
- 0.5
- 0.5
- 0.5
- 0.5
- 0.2

CV (%) for y:
- 39
- 33
- 24
- 1.8
- 0.9
- 0.2
- 0.1
- 0

Fig. 6. Correlations between the calibration constant for autochthonous production (k) and (a) the mean annual epilimnetic temperature (MAET), (b) the dynamic ratio (DR = \sqrt{A/D_m}), and (c) total phosphorus concentration (TP).
In the first basin of Lake Balaton, there is a small peak in SPM concentration in April, which is hinted also in the model prediction. This peak coincides with a peak in total phosphorus concentration, which is also true for the July and September peaks in Basin 2. A large peak in phosphorus concentration in December does not result in high SPM concentration, which is explained by the unfavorable temperature and light conditions. The SPM concentration in Lake Balaton is extraordinary variable, and the dynamics are not very well predicted by the model. The dynamic behavior of the lake is probably related to primary production rather than inflow or resuspension. The mean monthly wind speed varies between 3.2 and 4.2 m/s, with the highest values during the summer. High resuspension rates are generally associated with storm events, more or less evenly distributed during the year and not well represented by the mean monthly wind speed or by this model. The empirical description of the lake behavior should be regarded uncertain (see Fig. 4b).

The value of the lake specific calibration constant for primary production (\(k\)) used in this model may reveal some information about the lake ecosystems. In Fig. 6 the correlation between the obtained values of this constant and some other lake parameters is shown. It is seen that \(k\) correlates well with the dynamic ratio and with the total phosphorus concentration. There are uncertainties with the quantitative prediction of autochthonous SPM and this is an area for further research. A more extensive dataset from a wide range of lakes could improve this analysis.

Apart from the uncertainties associated with the autochthonous and autochthonous supply of SPM, there may be other reasons for the discrepancies between modeled and empirical values. It is not unlikely that the settling velocity should be different for autochthonous and allochthonous material. Phytoplankton and zooplankton are likely to have longer retention times in the water column than, e.g. humic material.

6. Conclusions

The main conclusions from this work are:

1. The internal processes have been quantified in an acceptable manner, as shown by the generally good agreement between predicted values and empirical data, although the empirical mean values are based on rather few measurements giving wide uncertainty bands around the curves. The sensitivity analyses show that the uncertainties associated with internal processes are less important.

2. Empirical data should be used for allochthonous inflow of SPM. The results presented by Håkanson et al. (2003) show that the CV for SPM in rivers is 1.71, and this very high CV indicates that no models can be expected to predict SPM in rivers very well.

3. The most important model uncertainty in these lakes is the autochthonous production. It is at present even more difficult to predict the autochthonous production than the allochthonous inflow. However, given the characteristic natural features of the primary production this key factor may be more predictable than the inflow (see Håkanson and Boulion, 2002).

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References


