A dynamic model for suspended particulate matter (SPM) in rivers

Lars Håkanson

ABSTRACT

Aim To present a general, process-based river model for suspended particulate matter (SPM).

Location General approach based on processes; data from Europe and Israel.

Methods The model has been tested and calibrated using an empirical river model for SPM and validated (blind-tested) using data from seven European sites. This modelling gives mean monthly SPM concentrations in water for defined river sites. The model is based on processes in the entire upstream river stretch (and not for given river segments) and calculates the transport of SPM from land to water, primary production of SPM (within the upstream river stretch), resuspension, mineralization and retention of SPM in the upstream river stretch (but not bed load of friction materials, such as sand). The catchment area is differentiated into inflow (~ dry land) areas and outflow area (~ wetland areas dominated by relatively fast horizontal SPM-fluxes). The model is simple to apply in practice as all driving variables may be accessed readily from maps. The driving variables are: latitude, altitude, continentality, catchment area and mean annual precipitation.

Results Modeled values have been compared to independent empirical data from sites covering a relatively wide domain (catchment areas from 93 to 5250 km², precipitation from 400 to 660 mm year⁻¹, altitudes from −210 to 150 m.a.s.l., latitudes from 47 to 59° N and continentality from 200 to 1000 km from the ocean). When blind-tested, the model predicts annual SPM-fluxes well.

Conclusion When modelled values are compared to empirical data, the slope is almost perfect (1.03) and the r²-value is 0.9996. This is good, given the fact that there are several simplifications in the model structure. It must, however, be stressed that there are only seven validation cases and that this model has not been tested for small catchments.

Keywords Dynamic model, Europe, Israel, predictive power, models, rivers, SPM, suspended particulate matter, validation.

INTRODUCTION

Suspended particulate matter (SPM) regulates the transport of all types of water pollutants in dissolved and particulate phases in lakes, rivers and coastal areas; it regulates water clarity and the depth of the photic zone, and hence also primary and secondary production; it regulates bacterioplankton production and biomass, and hence also mineralization, oxygen consumption and oxygen concentrations; and it regulates sedimentation, and hence also the use of sediments as an historical archive, e.g. of water pollutants (Håkanson, 2005a). The aim of the dynamic river model for SPM presented in this paper is to structure existing knowledge on the factors regulating variations among and within rivers (on a monthly basis) of SPM in a rational and quantitative manner.

Many papers and books discuss relationships between water discharge and SPM in rivers (see Jansson, 1982; Walling & Amos, 1999; Walling, 2000), but few models for SPM in rivers meet criteria for general applicability, easy access of the driving variables, high predictive power and suitability as a submodel in more
general models for water pollutants. There exist many physical/hydraulic models for SPM in rivers, which often omit biota and quantify SPM-fluxes at small time scales (minutes to days), small spatial scales (using partial differential equations) and utilize online climatological data. Such models will not be discussed in this work, which focuses on characteristic SPM-values in entire river stretches at monthly time scales.

Many factors are known to influence SPM in aquatic systems (Vollenweider, 1958, 1960; Carlson, 1977, 1980; Brezonik, 1978; OECD, 1982; Ostapenia et al., 1985; Preissendorfer, 1986; Bouillon, 1994, 1997; Wetzel, 2001). The most important factors are (i) autochthonous production (i.e. the amount of plankton, faeces, etc. in the water — more plankton means a higher SPM), (ii) allochthonous materials, such as the amount of coloured substances, and (iii) the amount of resuspended material. This is easy to state qualitatively, but more difficult to express quantitatively because these factors are not independent: high sedimentation leads to high amounts of resuspendable materials; high resuspension leads to high internal loading of nutrients and increased production; a high amount of coloured substances means a smaller photic zone and a lower production; a high production would mean a high sedimentation. SPM is generally a complex mixture of substances of different origins with different properties (size, form, density, specific surface area, capacity to bind pollutants, etc.). SPM may be divided into an organic fraction (POM) and an inorganic one (PIM, particulate inorganic materials). Total organic matter (TOM) is generally divided into particulate (POM) and dissolved (DOM) fractions. Normally, POM is about 20% of TOM, but this certainly varies among and within systems (Ostapenia, 1989; Velimorov, 1991; Bouillon, 1994). Normally, about 4% of POM is living matter and the rest is dead organic matter (detritus). About 80% of TOM is generally in the dissolved phase, and of this about 70% is conservative in the sense that it does not change due to chemical and biological reactions in the water (Ostapenia, 1987, 1989).

The physical features of rivers change along their longitudinal axis in a predictable manner. According to the river continuum concept, channels increase in width and length. The literature on river ecosystems is extensive (Huet, 1949; Illies, 1961; Vannote et al., 1980; Ward, 1989, 1998; Calow & Petts, 1994; Ward & Stanford, 1995; Bloesch, 1997). There are typical longitudinal changes in water discharge, water velocity, SPM-concentration and grain size characteristics of the sediments on the river bed; also the biota changes along the river stretch. Mountain streams are often dominated by salmonides whereas, e.g. cyprinids are often caught in lowland rivers. Algorithms for longitudinal changes in river morphology will be presented in this paper.

The characteristic coefficient of variation (CV) for within-site variations of SPM in rivers is 1.71 (= 171%; Håkanson et al., 2005), a very high value. This will restrict the predictive power of any model targeting SPM in rivers, and any model that uses SPM in rivers as an x-variable to predict a given y-variable.

During the last 10 years, there has been something of a ‘revolution’ in aquatic ecosystem modelling. The major reason for this development is, in fact, the Chernobyl accident. To follow the pulse of radionuclides through ecosystem pathways has meant that important transport routes have been revealed and the algorithms to quantify them developed and tested (Håkanson, 2000). It is important to stress that many of those structures and equations are valid not just for radionuclides, but for most types of contaminants, e.g. for metals, nutrients and organics — and for SPM — in most types of aquatic environments (rivers, lakes and coastal areas).

The dynamic river model presented in this work is meant to predict SPM in water based on processes and mechanistic principles. The model should also be as small as possible and the obligatory driving variables should be readily accessed from standard monitoring programmes and/or maps. The tests of the model will use empirical data on the tributary inflow to lakes discussed by Håkanson (2005a; three Belarusian lakes, Lake Erken, Lake Kinneret and Lake Balaton; see Table 1), and the calibrations will use the empirical SPM-model presented by Håkanson et al. (2005); see Fig. 1. The dynamic river model is based partly on the lake model discussed by Håkanson (2005a) but also on the catchment and river models for toxic substances, which have been tested with good results for radiothorium and radiothorium (Håkanson, 2004a,b, 2005b). The following section will first discuss the modifications, as compared to the lake model and the previous river model for toxic substances; the second part gives results from calibrations and sensitivity tests of the new dynamic river model; and the third part presents results from the validations. It should be noted that the model discussed here is intended to be a generic SPM-model for defined river stretches (and not for river segments). It is intended for river stretches with catchment areas larger than about 100 km².

METHODS

The total amount of substances in the water is often separated into a particulate phase, the only phase subject to gravitational sedimentation, and a dissolved phase. Operationally, the limit is generally determined by means of filtration using a pore size of 0.45 μm. Evidently, this is an operational approach and many colloidal particles will pass such filters (Bouillon, 1994) and are, hence, operationally included in the dissolved fraction, although they are not truly dissolved in a chemical or biological sense. SPM as determined in this way by filtration, drying and weighing is sometimes (Gray et al., 2000) also referred to as SSC, the suspended sediment concentration. Filtration is often a justifiable method from ecological and mass-balance modelling perspectives.

The models presented in this work have been produced using Stella software.

Data used to test the model critically

These data emanate from river stations related to the following lakes (see Table 1).

1 The Belarusian lakes Batorino, Miastro and Naroch; basic data from several sources compiled by Professor Alexander Ostapenia, Belarus State University, Minsk; the data from the 1990s have been used by e.g. Håkanson & Bouillon (2002).
Table 1 Characteristics of the lakes used to test the dynamic SPM-model. A gives lake data and B gives data on tributary SPM-transport to the lakes from the empirical measurements

<table>
<thead>
<tr>
<th>A: Lake data</th>
<th>Latitude °N</th>
<th>Altitude m a.s.l.</th>
<th>Continentality km</th>
<th>Annual prec. mm year⁻¹</th>
<th>Catchment km²</th>
<th>Lake area km²</th>
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<td>150</td>
<td>500</td>
<td>650</td>
<td>93</td>
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<tr>
<td>Mastro, Belarus</td>
<td>55</td>
<td>150</td>
<td>500</td>
<td>650</td>
<td>130</td>
<td>13</td>
</tr>
<tr>
<td>Naroch, Belarus</td>
<td>55</td>
<td>150</td>
<td>500</td>
<td>650</td>
<td>280</td>
<td>80</td>
</tr>
<tr>
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<td>210</td>
<td>200</td>
<td>408</td>
<td>2560</td>
<td>168</td>
</tr>
<tr>
<td>Erken, Sweden</td>
<td>59</td>
<td>11</td>
<td>500</td>
<td>660</td>
<td>141</td>
<td>24</td>
</tr>
<tr>
<td>Balaton, Hungary</td>
<td>47</td>
<td>106</td>
<td>1000</td>
<td>600</td>
<td>5280</td>
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<td>600</td>
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</tbody>
</table>

SPM-inflow (t year⁻¹) calculated from:

<table>
<thead>
<tr>
<th>B: SPM-data</th>
<th>Emp. river model</th>
<th>Dyn. river model</th>
<th>Emp. data, lake model</th>
</tr>
</thead>
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<tr>
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<tr>
<td>Balaton 1</td>
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</tr>
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</table>

Figure 1 Illustration of the empirical submodel to estimate concentrations of suspended particulate matter (SPM) from latitude (Lat), mean annual water discharge (Q) and continentality (Cont). The model comes from Häkanson et al. (2005). The submodel for water discharge (Q) comes from Abrahamsson & Häkanson (1998).

Figure 2 Illustration of the river model with inflow and outflow areas (from the catchment) and the compartments for river water and ET-areas characterizing the upstream river stretch (OURS).

2 The data from Lake Kinneret, Israel, have been compiled by Dr Arkadi Parparov, Kinneret Limnological Laboratory, Tiberias; these data have been used by e.g. Häkanson et al. (2000). The data used here represent conditions for the period 1992–96.

3 The data from Lake Erken, Sweden, originate from the Erken Laboratory, Uppsala University (and Professor Kurt Pettersson). They represent conditions during the 1990s.

4 The data from Lake Balaton, Hungary, come from Professor Vera Istvanovics, Budapest University of Technology; they have also been used by Malmaeus & Häkanson (2003) and represent conditions between 1989 and 1999.

Modifications of the lake model

Figure 2 gives a schematic description of the modelling approach. There are major differences between lakes and rivers, and one fundamental difference concerns the relationship between the catchment area, the water surface area of the river upstream at a given site and the water discharge at this site. The ratio between the drainage area (A_dr) and the lake area can vary from about 2 to over
L. Håkanson

2000, whereas the river water discharge (Q) is directly related to catchment area (Q = SR × A_Catch, where SR is the specific runoff rate in m³ m⁻² × time⁻¹). An evident change compared to the lake model is to define the upstream river stretch and its geometry.

1 The algorithm in the lake model to calculate the fraction of bottom areas where erosion and transport processes of fine sediments occur (the ET-areas) has been replaced by a general constant, ET = 0.95, for all river stretches (in the domain of this model, i.e. for relatively large catchments). In all rivers, there are topographically sheltered parts, macrophyte beds or deep holes where SPM may be retained.

2 The algorithm to calculate deep-water temperatures has been omitted because the river water is not treated as being stratified. Therefore, there is only one river-water compartment (RW). This also implies that upward and downward mixing are omitted.

3 The algorithm to calculate sedimentation in lakes has been modified because the river flow velocity regulates the turbulence of the river water and hence also sedimentation and resuspension of SPM. This will be quantified by a dimensionless moderator (Ys), which also replaces the algorithm in the lake model that relates the settling velocity of SPM to the resuspended fraction and the age of the ET-sediments.

4 The distribution coefficient (Vd/3; Vd is the form factor = 3 × Dsw/Dsw; Dsw = the mean depth; Dsw = the maximum depth), which distributes the resuspended matter from ET-areas either to surface-water areas or to deep-water areas in the lake model, has also been omitted. In the river model, there is resuspension only from ET-areas to river water (RW).

5 The geometry (mean depth, mean width and length) of the entire upstream river stretch is estimated by simple equations. Evidently, if empirical data characterizing the geometry of the upstream river stretch (URS) are available, such data can preferably be used. The algorithms to estimate river geometry have, however, been used in all the following calibrations and validations.

The next section presents the catchment area submodel. The catchment is divided into: (i) outflow areas (OA – wetlands) dominated by a relatively fast turnover of substances and horizontal (land overflow) transport processes (Eriksson, 1974; Nystrom, 1985; Rodhe, 1987); and (ii) inflow areas (IA – dry land) dominated by vertical transport processes, first through soil horizons, then ground water and, finally, transport to the river.

THE DYNAMIC RIVER MODEL

This section will first present the submodel to estimate the geometry of the upstream river stretch, then the submodel quantifying SPM transport from land to river water and, finally, the model for the upstream river stretch.

The geometry of the upstream river stretch

These calculations are performed in steps and one can start with the expression by which the total length of the upstream river stretch (L) is estimated from the catchment area (A_Catch). Just as the diameter of a circle is related to the square-root of the area, the river length is estimated from the square-root of A_Catch. However, the river length should be shorter at high altitudes and high latitudes and longer for very continental rivers (Strahler, 1963). How differences in altitude, latitude and continentality influence L will be expressed by three dimensionless moderators, which also have been used to regulate the specific runoff rate of SPM from catchments. Many approaches to define the moderators have been tested and some of those tests will be given in the following section. The length of the upstream river stretch (L in m) is first estimated from:

\[ L = 10 \times Y_{A_DA} \times Y_{lat} \times Y_{alt} \times Y_{cont} \]  \hspace{1cm} (1)

where \( Y_{A_DA} \) (in m) is simply given by \( \sqrt{\frac{A_{DA}}{L}} = \frac{A_{DA}}{\sqrt{L}} \) (A_in m²). Ylat is the dimensionless moderator expressing how changes in latitude probably influence L, where:

\[ Y_{lat} = [75/(\text{Lat} - 35)]. \]  \hspace{1cm} (2)

This moderator is meant to apply for rivers at latitudes between 75 and 40° N. Ylat attains values between 15 and 75, which means that L (i.e. the total length of rivers in the entire upstream river stretch) can be expected to be about five times longer at a latitude of 40° N than at 75° N — if all else is constant. In the following calibrations, the default latitude was set to 50° N, which means that Ylat = 5. The moderator for altitude is defined by:

\[ Y_{alt} = [(1/(\text{Alt} + 25))]^{0.5}. \]  \hspace{1cm} (3)

In the following calibrations, the default altitude is set to 0 m a.s.l., which gives Yalt = 0.2. If altitude is 1000 m a.s.l., Yalt = 0.03 and the river about 6.7 times shorter, compared to at altitude = 0. Continentality will influence L in the following manner:

If Cont < 500 km then \( Y_{cont} = 1 \) else \( Y_{cont} = (\text{Cont/500})^{0.5}. \)  \hspace{1cm} (4)

This means that a continentality less than 500 km will not influence the L-value; if Cont = 1000 km, Ycont = 1.4; if Ycont = 5000 km, Ycont = 3.2, etc. So, continentality influences L less than altitude and latitude. The default continentality for the following calibrations was set to 100 km.

Note that these rules apply for the entire upstream river stretch (and not for river segments) and they have been used in the following model tests. Figure 3 illustrates how the algorithms work. Figure 3(a) shows the relationship between the length of the upstream river stretch (L) and the area of the catchment area (A_DA) for river sites at different latitudes, Fig. 3(b) shows the same thing for sites at different altitudes and Fig. 3(c) the corresponding nomograms for different continentals.

The mean depth of the upstream river stretch (Dsw in m) and the mean flow velocity in the upstream river stretch (v in m s⁻¹) are estimated from mean monthly water discharge (Q; Fig. 4a,c); the mean river width (B in m), in turn, is estimated from mean depth (Dsw; Fig. 4b) using algorithms presented and tested by Håkanson (2005b). The volume of the upstream river stretch is calculated from mean depth (Dsw) and the area of the upstream river stretch (Area = B-L in m²). The volume is needed to calculate river water concentration of SPM (C_RW = M_RW/V). The flow
velocity \( (v) \) is needed both in the new algorithm for sedimentation and in the new algorithm for resuspension. There should be little sedimentation if \( v \) is higher than 0.5 m s\(^{-1}\) and better possibilities for sedimentation of suspended particles if \( v \) is smaller than 10 m s\(^{-1}\) (Postma, 1967, 1982). This is used to define the dimensionless moderator expressing how flow velocity \( (v) \) influences sedimentation and resuspension \( (Y_s) \).

Figure 4 gives a compilation of the submodel used to calculate the geometry of the upstream river stretch.

The catchment area submodel

This submodel (Fig. 6) quantifies SPM-fluxes from land to water. Compared to a previous catchment area submodel for dynamic model for SPM in rivers.
Figure 5 Outline of the submodel defining the geometry of the upstream river stretch (length, mean depth, water flow velocity, water discharge, etc.) and how the geometry depends on latitude, area of catchment area and altitude.

Figure 6 Outline of the catchment area submodel. The calibrations of the dynamic river model have focused on the value for the default runoff rate ($R_{defOA}$ in g m$^{-2}$ × month) and the characteristics of the dimensionless moderators for how variations in latitude, altitude, mean annual precipitation and continentality influence the runoff of SPM from land to river water.
Figure 7 The relationship between catchment area ($A_{\text{CA}}$) and outflow areas (OA = wetland areas) used in this dynamic river model. The figure also gives data of OA and $A_{\text{CA}}$ from Häkanson (2004a,b). radionuclides (Häkanson, 2004a,b), this model is modified in several ways. The requirement to include the fraction of outflow areas (OA) as an obligatory driving variable has been omitted, and OA is now predicted from first catchment area ($A_{\text{CA}}$; Fig. 7) and the value also depends on latitude and altitude. The main reason for using this algorithm is that it is sometimes difficult to obtain reliable data on OA, especially for large and topographically complex catchments. $A_{\text{CA}}$, latitude and altitude, on the other hand, can be determined easily and accurately. Note that the equation given in Fig. 7 to estimate OA from $A_{\text{CA}}$ is not a regression. It is a deterministic relationship based on the boundary requirements that OA should be about 0.25 for small catchments (~0.2 km$^2$) and approach 0.02 for large catchments (Häkanson & Peters, 1995). The equation to predict OA from $A_{\text{CA}}$ has been tested against empirical data and it describes the data reasonably well (Fig. 7), so it has been used in this river model. Evidently, if reliable empirical data on OA are available, it is preferable to use such data rather than this submodel. From the relationship given in Fig. 7, OA may be estimated by:

$$OA = \left(\frac{\text{Lat}}{60}\right) \times \left[1 + 0.0025 \times (\text{Alt}/1 - 1)\right] \times 10^{0.19 \log(\text{Alt}) - 0.71}$$

where (Lat/60) is a simple dimensionless moderator which gives that OA will increase with latitude (see Fig. 8b). The moderator for altitude is given by:

$$Y_{\text{OA}} = \left[1 + 0.0025 \times (\text{Alt}/1 - 1)\right],$$

which means that (Fig. 8a) OA will increase at higher altitudes; if Alt = 0, $Y_{\text{OA}}$ = about 1 and if Alt = 1000 m a.s.l., $Y_{\text{OA}}$ = 3.5 and OA is 3.5 times larger than at the sea level.

An important part of the catchment area submodel is to predict river discharge (the Q-model; Fig. 9) and monthly variations in river discharge (as given by the dimensionless moderator for Q, $Y_Q$). Differences in hydrological regimes affect the transport of substances from catchments (Brittain et al., 1994). In spite of the fact that river discharge is a variable with great temporal and spatial variability, this submodel to predict river discharge has yielded good predictions ($r^2 = 0.84$, $n = 119$; see Abrahamssson & Häkanson, 1998). The best results were achieved for rivers with a mean annual discharge in the range 1–500 m$^3$ s$^{-1}$.

The monthly flow of SPM from inflow areas (IA) to the upstream river stretch ($F_{\text{upr}}$, in g month$^{-1}$) can then be calculated from:

$$F_{\text{upr}} = [A_{\text{CA}} \times (1 - OA) \times Y_Q \times R_{\text{OA}}]/12$$

where $A_{\text{CA}}$ = the catchment area (m$^2$); OA = the fraction of outflow areas [IA = (1 - OA), dimensionless]; $Y_Q$ = the dimensionless moderator for water discharge giving mean monthly Q-values (m$^3$ month$^{-1}$) from mean annual values (divided by 12); and $R_{\text{OA}}$ = the specific runoff rate for SPM (g SPM$^{-1}$ m$^{-2}$ month$^{-1}$). The specific runoff rate depends on latitude, altitude, continentality and precipitations. This is given by:

$$R_{\text{OA}} = R_{\text{OA}}Y_{\text{pre}}Y_LY_{\text{OA}}Y_{\text{sm}}$$

The calibrations have focused on the value for the default specific runoff rate ($R_{\text{OA}}$ in g SPM$^{-1}$ m$^{-2}$ month$^{-1}$); the dimensionless
Equations:
\[ Y_Q = 1 + 0.526 \times (\text{Lat} - 35)^2 + 0.218 \times \text{Li} + (1 - \text{Lat} - 35)^2 + 0.218 \times \text{Li}) + 0.421 \times (\text{Ax} - \text{Alt}^0.51 / 1000^0.51 + \\
A_t (1 - \text{Alt}^0.51 / 1000^0.51) + 0.265 \times (Q_x Q_0.22 / 50000^0.22 + Q_t (1 - Q_0.22 / 50000^0.22)); \\
Q = 0.01 \times (\text{Prec} / 650) - \text{AD}, (\text{m}^3 / \text{sec})
\]

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<th>Month</th>
<th>Qmax norm</th>
<th>Qmin norm</th>
<th>Latmax norm</th>
<th>Latmin norm</th>
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Figure 9 The submodel for tributary water discharge (compiled from Abrahamsson & Håkanson, 1998).

moderators for latitude (Y latitude), altitude (Y altitude) and continentality (Y continentality) have already been defined. The specific runoff rate of SPM will be higher at lower latitudes and altitudes and higher if the continentality is higher than 500 km. The dimensionless moderator for precipitation (Y precipitation) comes from the river model for radionuclides (Håkanson, 2004a,b) and gives:

\[ Y_{\text{prec}} = (\text{Prec} / 650)^2. \]

(9)

There is an exponential and not a linear increase in SPM runoff (the exponent is 2) and the normal mean annual precipitation is set to 650 mm year\(^{-1}\) in this moderator.

The monthly flow of SPM from outflow areas to the upstream river stretch \(F_{\text{OAR}}\) in g month\(^{-1}\) is calculated in a similar way. The main difference is that the specific runoff rate should be higher and in this modelling it is set to 10 times higher than the value defined for the inflow areas. This gives:

\[ F_{\text{OAR}} = (10 \times A_{\text{BA}} \times OA \times Y_{\text{OA}} \times R_{\text{OA}}) / 2. \]

(10)

Figure 10 illustrates the calibration routine and also how the default value for the specific runoff rate \(R_{\text{BA}} = 2\) g m\(^{-2}\) month\(^{-1}\) has been obtained. The calibrations have been performed along gradients, in this case along a catchment size gradient \(A_{\text{BA}}\) set to 10, 100, 1000 and 10,000 km\(^2\) in four steps in Fig. 10a; this is the driving variable in this test). Three different equations for the dimensionless moderator for how \(A_{\text{BA}}\) influences SPM have been tested. Figure 10b also gives the data, as calculated using the empirical SPM-model, along this gradient. The idea has been to see if there is a logical relationship between the SPM-values given by the empirical model (Fig. 2) and the dynamic model. From Fig. 10b, one can note: (i) the good correspondence between SPM-values predicted by the empirical and the dynamic models for small catchment areas (when \(A_{\text{BA}}\) is set to 100 km\(^2\)), but the poorer correspondence for the larger \(A_{\text{BA}}\)-values; (ii) the higher the value for the default runoff rate \(R_{\text{BA}}\), the higher the SPM-inflow from inflow and outflow areas (Fig. 10c); (iii) Fig. 10c also gives the calculated changes in river discharge (monthly Q in m\(^3\) s\(^{-1}\)) related to these changes in \(A_{\text{BA}}\) (Fig. 10a); (iv) to understand the difference between the predictions from the empirical model and the dynamic model in Fig. 10b, one must also look at the changes in river geometry associated with the given changes in \(A_{\text{BA}}\); figure 10d shows how river length (L) and mean depth of the upstream river stretch \(D_{\text{UP}}\) increase with increasing \(A_{\text{BA}}\) and (v) that this is also a sensitivity test where the \(A_{\text{BA}}\) has been varied while all else, including precipitation, have been kept constant.
One can conclude that the empirical model predicts SPM from changes in water discharge (Q) at a given river site, but Q is basically calculated from $A_{10}$ and precipitation (as $Q_{\text{ave}} = 0.01 \times A_{10} \times \text{Prec}/650$, where $Q_{\text{ave}}$ is the mean annual Q in m$^3$ s$^{-1}$, 0.01 is the default value of the specific runoff rate for water in m$^3$ km$^{-2} \times$ s$^{-1}$ and Prec is the mean annual precipitation in mm year$^{-1}$). Figure 11(b) shows a similar comparison between the empirical SPM-model and the dynamic model along a precipitation gradient (Fig. 11a; mean annual precipitation set to 400, 600, 800 and 1000 mm year$^{-1}$). In this test, the exponent in the dimensionless moderator for precipitation has been varied in three steps (1, 2 and 3). One can note that the default exponent of 2 provides good correspondence with the empirical SPM-model along the precipitation gradient, although the values from the dynamic model are initially low; for higher precipitation, the predicted SPM-values are higher than those given by the empirical model. Figure 11(c) illustrates how the SPM-transport from inflow and outflow areas depend on variations in precipitation (using the default values, e.g. the exponent in the precipitation algorithm = 2).

With the dynamic model, one can quantify the fluxes causing the given SPM-concentrations. The basic idea is that the two models should give similar results when tested on real systems. These initial calibrations and sensitivity tests are meant to illustrate the difference between the two modelling approaches. In the dynamic model, the changes in catchment area influence the geometry of the upstream river stretch, and hence also the outflow from the upstream river stretch. The different roles that variations in catchment area and precipitation have may be studied using the dynamic model, not just for the SPM in river water but also for the transport processes, i.e. the flow from inflow and outflow areas, outflow, sedimentation, resuspension, mineralization in river water and on ET-areas and primary production in the upstream river stretch.

The submodel for the upstream river stretches (URS)

The URS submodel (see Fig. 12, which shows the entire model, and Table 2, which gives a compilation of all key equations) handles input from the catchment area (one flux from outflow areas to the river water, $F_{\text{URS}}$), another flux from inflow areas, $F_{\text{inflow}}$, internal fluxes (sedimentation on ET-areas and A-areas, i.e. areas of fine sediment erosion plus transportation and accumulation, respectively), primary production of SPM in the upstream river stretch ($F_{\text{pprod}}$; see eqn 11), resuspension from ET-areas back to river water ($F_{\text{ETRAW}}$), mineralization, i.e. losses of SPM from bacterial decomposition in river water ($F_{\text{min}}$) and in sediments on ET-areas ($F_{\text{minET}}$) and outflow to downstream river stretches ($F_{\text{out}}$).

Calculating bioproduction is a focal issue in limnology. Generally, mean summer chlorophyll-a concentrations are predicted from water temperature, light conditions and nutrient status (e.g. Dillon & Rigler, 1974; Smith, 1979; Riley & Prepas, 1985; Evans et al., 1996). The equation to quantify the amount of SPM...
Figure 11 Calibrations and sensitivity analyses of the river model along a gradient in precipitation: (a) values set to 400, 600, 800 and 1000 mm year⁻¹ using the three different equations for the dimensionless moderator for how precipitation influences SPM-concentrations in river water; (b) gives the data calculated using the empirical SPM-model along this gradient; and (c) shows how this will influence the transport of SPM from inflow and outflow areas.

generated on a monthly basis in the upstream river stretch used here comes from Håkanson & Bouillon (2002). In this approach, total SPM-production is calculated from chlorophyll-a, accordingly:

\[
F_{\text{prod}} = (30.6 \times \text{Chl}^{0.927} \times 0.45 \times 30 \times \text{Area} \times \text{Sec} \times 0.001 \times [(\text{RWT} + 0.1)/9] \times (\text{BM}_{\text{pl}}/\text{BM}_{\text{pl1}}).
\]  

(11)

Chl = the mean monthly chlorophyll concentration (μg l⁻¹); the expression \(30.6 \times \text{Chl}^{0.927}\) transforms Chl into phytoplankton production (in μg C l⁻¹ d⁻¹). The factor 0.45 is a standard transformation factor to change g C to g dw. Multiplication with 0.001, 30 days, area and the mean monthly value of the effective depth of the photic zone (= the Secchi depth = Sec in m) gives the biomass of phytoplankton produced per month (g dw per month).

\[
\text{BM}_{\text{pl1}}/\text{BM}_{\text{pl}} = \text{the ratio between the biomass of all sorts of plankton (phytoplankton, bacterioplankton zooplankton, etc.; BM_{pl}) to the calculated biomass of phytoplankton (BM_{pl1}) (Håkanson & Bouillon, 2002). This ratio is, on average, about 2.5. It indicates that the total biomass of bacterioplankton plus zooplankton plus phytoplankton is a factor of about 2.5 higher than the phytoplankton biomass. There is a marked temporal variability around this mean value from seasonal changes in water temperatures. This is quantified by the dimensionless moderator for RWT in eqn 11.}
\]

\[
\text{RWT = mean monthly river-water temperatures (°C), e.g. calculated from the temperature submodel given by Ottosson & Abrahamsson (1998). By dividing RWT with a reference temperature of 9 °C (related to the duration of the growing season), this approach accounts for seasonal variations in RWT in a dimensionless manner. The moderator is (RWT + 0.1)/9. The constant 0.1 is used as RWT may approach 0 °C during the winter and as there is also production under the ice.}
\]

\[
\text{Sec = Secchi depth (in m), measured either using Secchi discs or calculated from empirically or dynamically modelled values of SPM (mg l⁻¹). This modelling will generally use values of Secchi depth calculated from dynamically modelled SPM-values. The regression (from Håkanson, 2005a) is given by:}
\]

\[
\text{Sec} = 10^{(-0.27 \text{log} \text{SPM} + 0.77)}.
\]

(12)

If no empirical data on chlorophyll are available, the first term in eqn 11 may be estimated in the following way:

\[
(30.6 \times \text{Chl}^{0.927}) = (30.6 \times \text{Chl}_{\text{pl1}}^{0.927} \times (A_{\text{pl}}/10,000)^{0.7},
\]

(13)

where \(\text{Chl}_{\text{pl1}}\) is the default (= reference) values for mean monthly chlorophyll (in μg l⁻¹); the following values from the River Danube at a site near Regensburg have been used in all the following simulations; from Håkanson et al., 2003):

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<tr>
<td>2.6</td>
<td>9.6</td>
<td>9.1</td>
<td>26.7</td>
<td>33.6</td>
<td>21.9</td>
<td>21.8</td>
<td>19.6</td>
<td>6.7</td>
<td>6.6</td>
<td>3.8</td>
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The ratio between \(A_{\text{pl}}\) (in km²) and the size of the reference site (area = 10,000 km²) illustrates that the requested chlorophyll-values should be higher in rivers with larger catchments than in rivers with small catchments. The exponent 0.7 indicates that there is not a linear increase in SPM-production with \(A_{\text{pl}}\). This approach is evidently a simplification. It means that if \(A_{\text{pl}} = 1000\) km², if \(\text{Chl}_{\text{pl1}} = 25\) μg l⁻¹ (as it is for the reference site for the growing season), then eqn 13 gives 600 × 0.2 (μg C l⁻¹ day⁻¹); if \(A_{\text{pl}} = 10,000\) km², the left side is equal to the right side = 600; if \(A_{\text{pl}} = 100,000\) km², the value is 600.5. The other factors in the production algorithm \(0.45 \times 30 \times \text{Area} \times \text{Sec} \times 0.001 \times [(\text{SWT} + 0.1)/9] \times (\text{BM}_{\text{pl1}}/\text{BM}_{\text{pl}})\) are the same for the lake model and the river model.

Sedimentation is calculated in a modified way accounting for river flow velocity and the fact that little fine sediments will settle if the velocity is higher than about 10 cm s⁻¹; if the velocity is higher than 50 cm s⁻¹ there is probably no net sedimentation but
rather erosion and transportation of all types of fine particles. This means that the sedimentation rate ($R_{sed}$ in 1 month$^{-1}$) is given by the following approach:

$$R_{sed} = (72/12) \times \left(1/D_{50}\right) \times Y_{SPM} \times Y_r$$

where the default settling velocity for SPM is the same as in the lake model ($72/12 = 6$ m month$^{-1}$). The main difference between the lake model and the river model concerns the dimensionless moderator for the flow velocity ($Y_r$), which influences both sedimentation and resuspension.

If $v < 0.1$ m s$^{-1}$ then $Y_r = 1$ else $Y_r = [1 - 0.25 \times (v/0.1 - 1)]$; if $Y_r < 0.01$ then $Y_r = 0.01$

The resuspension rate ($R_{res}$) is given by:

$$R_{res} = (1 - Y_r)$$

Mineralization means net losses of SPM mainly from bacterial decomposition of the organic fraction of SPM. The value used for the mineralization rate, $R_{min}$, regulates the total amount of SPM being lost each month. The mineralization rate operates in this model on SPM in the river water and on the sediment compartment for ET-areas. Mineralization is further assumed to be proportional to temperature (RWT = river-water temperature in °C). The loss of SPM from mineralization in river water is:

$$F_{minRW} = M_{RW} \times R_{res} \times Y_{ET} \times (RWT/9)^{1/2},$$

where 9 °C is a reference temperature related to duration of the growing season (Håkanson & Boullion, 2002); the default value for the mineralization rate ($R_{min}$) is set to 0.125 (per month; Håkanson et al., 2000); and the mass of SPM in the river water ($M_{RW}$) is calculated automatically by the model. The ratio RWT/9 is used as a simple dimensionless moderator and the exponent 1.2 stresses the nonlinear temperature dependence of bacterial decomposition (Törnbom & Rydin, 1998). $Y_{ET}$ is a dimensionless moderator quantifying in a simple manner a more complicated phenomena related to the fact that resuspended particles are older and more likely to have been mineralized and have a lower organic content than primary particles (Håkanson,
Table 2 Basic equations making up the dynamic river model for SPM. M = mass; F = flow; R = rate; SWT = river water temperature; OA = outflow areas; Q = water discharge; ET = ET-areas; A = A-areas; RW = river water; Y = dimensionless moderator; AOA = area of drainage area; alt = altitude; cont = continentality; lat = latitude; prec = mean annual precipitation; Chl = chlorophyll-a; BM = biomass; flux from A to B = F_{AB} etc. Mass in RW = M_{RW}, etc.

Compartment, SPM in river water (M_{RW}):
M_{RW}(t) = M_{RW}(t - dt) + (F_{inflow} + F_{inlet} + F_{sed} + F_{ETRW} - F_{outflow} - F_{inlet} - F_{melt}) \times dt

INFLOWS:
\begin{align*}
F_{inflow} &= (30.6 \times \text{ch}_{\text{AD}}^0.6) \times (A_\text{OA}/10)^{0.7} \times 0.45 \times 30 \times \text{Area} \times 10^{0.7(\text{LogSPM}+1.77)} \times 0.001 \times ((\text{SWT} + 0.1)/9) \times (\text{BM}_{\text{AD}}/\text{BM}_{\text{in}}) \\
F_{inlet} &= (A_\text{OA} \times (1 - OA)) \times Y_i \times R_{\text{inlet}}/12 \\
F_{sed} &= 10 \times A_\text{OA} \times OA \times Y_i \times R_{\text{inlet}}/12 \\
F_{ETRW} &= M_{ET} \times R_{ET}
\end{align*}

OUTFLOWS:
\begin{align*}
F_{out} &= Q \times C_{\text{RW}} \\
F_{inlet} &= (1 - ET) \times M_{RW} \times R_{inlet} \\
F_{inlet} &= ET \times M_{RW} \times R_{inlet} \\
F_{melt} &= M_{RW} \times R_{melt}
\end{align*}

Compartment, SPM in ET-areas (M_{ET}):
M_{ET}(t) = M_{ET}(t - dt) + (F_{inlet} - F_{ETRW} - F_{melt}) \times dt

INFLOWS:
\begin{align*}
F_{ETRW} &= \text{see above} \\
F_{inlet} &= \text{see above} \\
F_{melt} &= M_{inlet} \times R_{melt}
\end{align*}

Some key rates, velocities and dimensionless moderators:

Resuspension rate = R_{inlet} = (1 - Y_i); if \nu < 0.1 m s^{-1} then Y_i = 1 else Y_i = (1 - 0.25 \times (\nu/0.1 - 1)); if \nu < 0.01 then 0.01

Runoff rate for OA = R_{OA} = 2 \times Y_{\text{prec}} \times Y_{\text{LS}} \times Y_{\text{L}} \times Y_{\text{con}}

Sedimentation rate = R_{sed} = (72/12) \times (1/D_{\text{AD}}) \times Y_{\text{SPM}} \times Y_{\text{cont}} [default settling velocity for SPM = 72/12 m month^{-1}]

Mineralization rate RW = R_{minRW} = R_{melt} \times (\text{SWT}/9)^{0.5}

Mineralization rate SPM = R_{minSPM} = 0.125 \times (0.99/ET) \times 0.125 \times (0.99/0.95) = 0.13

\nu = (0.5319Q^{0.2} + 0.0677) \times [Q \text{ in m}^3 \text{s}^{-1}; \nu \text{ in m s}^{-1}]

\begin{align*}
Y_{\text{AD}} &= \sqrt[4]{A_{\text{AD}}} \\
Y_{\text{inlet}} &= [1/(\text{Alt} + 25)]^{0.5} \\
Y_{\text{inlet}} &= [1 + 0.0025 \times (\text{Alt}/1 - 1)] \\
Y_{\text{cont}} &= \text{if Cont} < 500 \text{ km then } 1 \text{ else } \nu/(\text{Cont}/500) \\
Y_{\text{LS}} &= (75/(\text{Lat} - 35)) \\
Y_{\text{prec}} &= (\text{Prec}/650)^{0.5} \\
Y_{\text{SPM}} &= [1 + 0.75(\text{SPM}/50 - 1)]
\end{align*}

2005a). Generally, for aquatic systems, Y_{ET} is defined by the ratio (0.99/ET), but for rivers ET is set to 0.95 (as a general default value), so Y_{ET} = 0.99/0.95 = 1.04.

The outflow is given by:

\[ F_{out} = Q \times C_{\text{env}}. \] (18)

Q is the mean monthly water discharge at the given river site (in m³ month⁻¹) calculated from the submodel given in Fig. 9 and C_{env} is the SPM-concentration in the river water compartment (g⁻¹ m³), calculated from M_{inlet}/V; M_{inlet} = the mass (g); V is the volume of the upstream river stretch (m³).

The next section will give more calibrations of the dynamic river model and the aim is also to motivate the new algorithms and illustrate the model behaviour.

CALIBRATION OF THE MODEL

Figure 13 shows calibration results along a latitude gradient (latitudes set to 40°, 50°, 60° and 70° N in four steps; Fig. 13a1 gives the driving variable). Three different approaches for how latitude influences SPM-transport from catchment areas and river geometry (Y_{lat}) have been tested and compared to results using the empirical SPM-model as a reference. From Fig. 13a2, one can note the good correspondence between the SPM-values predicted by the empirical model and the dynamic model for the default moderator (curve 1), except that the dynamically modelled values are higher than the values given by the empirical model at low latitudes. The other two algorithms for Y_{lat} give lower values compared to the empirical SPM-model for rivers at low latitudes. The default latitude in these calibrations was set to 50° N. The next figure (Fig. 13b) gives calibrations along an altitude gradient (altitudes set to 0, 100, 500 and 1000 m a.s.l in four steps; Fig. 13b1 gives the driving variable; the default value in these calibrations is set to Alt = 0). Four moderators expressing how differences in altitude influence SPM-transport and river geometry (Y_{alt}) have been tested and compared to the empirical SPM-model. From Fig. 13b2, one can see: (i) that the empirical model does not account for differences in altitude, so there is just one reference line; (ii) that there
is a good correspondence between the SPM-values given by the empirical model and the dynamic model for the default moderator (curve 1); and (iii) that all moderators give higher SPM-values for low-altitude than for high altitude rivers, but the SPM-values vary greatly using the moderator expressed by curve 4. The last calibration is along a gradient in continentality (values set to 10, 100, 1000 and 10,000 km from the ocean in four steps; Fig. 13c1 gives the driving variable; the default value in these calibrations is set to Cont = 100 km). Three moderators expressing how continentality would influence SPM-transport from catchment areas and river geometry (Y_{cont}) have been compared to values given by the empirical SPM-model. One can note that there is an excellent correspondence between the values predicted by the two models for the default set-up (curve 2).

**RESULTS FROM VALIDATIONS**

From these results, one can ask how the dynamic model will work when validated, i.e. if the model predictions are blind-tested against independent data. The following validation will use data available to the author on SPM-inflow from tributaries to lakes (see Table 1a). First, it should be noted that these SPM-inflows are based on empirical data (either on SPM-inflow, as in Lake
Figure 14 Results of the validation of the dynamic river model for SPM. The figure gives the actual SPM-values predicted by the dynamic model on the x-axis compared to values calculated from empirical data on the river inflow of SPM to the seven lakes studied on the y-axis.

Kinneret, or on empirical data on TP-inflow recalculated as SPM-inflows as in Lake Erken, or on well-tested mass-balance calculations based on TP-inflow, as in the three Belarussian lakes and Lake Balaton and Lake Balaton 1; for more information, see Häkanson, 2005a). However, these data on SPM-inflow, although based on measurements and independent from the dynamic river model, are not ‘cut in stone’ but uncertain, at least by a factor of 1.5. Table 1b gives the results and Fig. 14 gives a direct comparison between the actual values from the dynamic river model (on the x-axis) and the empirically based inflow values to the seven lakes (on the y-axis). The correspondence is excellent \( r^2 = 0.9996, \text{slope} = 1.03 \), but it must be stressed that there are only seven cases.

CONCLUDING REMARKS

It must be emphasized that the good validation results for the dynamic river model should be taken with due reservation. These results, however, indicate that the mechanistic principles and assumptions behind the dynamic river model probably represent transport processes from land to water and within entire river stretches well for sites with catchment areas larger than about 100 km². For river sites with smaller catchments, one can assume that it will become important to account for differences in soil type, bedrocks, vegetation and land use. Evidently, it would be interesting to test the behaviour of the dynamic model for more rivers.

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BIOSKETCH

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